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## On Lyndon's equation in some $\Lambda$ -free groups and HNN extensions

L. Ciobanu, B. Fine and G. Rosenberger

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**Abstract.** In this paper we study Lyndon's equation  $x^p y^q z^r = 1$ , with  $x, y, z$  group elements and  $p, q, r$  positive integers, in HNN extensions of free and fully residually free groups, and draw some conclusions about its behavior in  $\Lambda$ -free groups.

### 1 Introduction

The classical result of Lyndon and Schützenberger ([9]) states that any elements  $x, y$  and  $z$  of a free group  $F$  that satisfy the relation  $x^p y^q = z^r$  for  $p, q, r \geq 2$  necessarily commute. In the paper of Brady, Ciobanu, Martino and O'Rourke ([1]) this result has been generalized to  $\Lambda$ -free groups. In particular, the following result has been obtained. Let  $G$  be a group that acts freely on a  $\Lambda$ -tree, where  $\Lambda$  is an ordered abelian group, and let  $x, y, z$  be elements in  $G$ . If  $x^p y^q = z^r$  with integers  $p, q, r \geq 4$ , then  $x, y$  and  $z$  must commute. It has been unclear whether the same conclusion holds for  $p, q, r$  not all larger than 4, and in particular the proof in [1] cannot be extended to these smaller integer cases. Here we shed light on the behavior of this equation in some HNN extensions and show that for  $p, q, r$  not all larger than 4 the conclusion of [1] does not always hold (see Corollary 1). This work complements the results in [5], where Lyndon's equation is studied in various amalgams of groups.

### 2 Results

**Theorem 1.** *Let  $F$  be a finitely generated non-cyclic free group, and let  $u$  and  $v$  be non-trivial elements in  $F$  which are not proper powers. Let  $G = \langle F, t \mid tut^{-1} = v \rangle$  and  $r \geq 2$  be a given integer. Then for particular choices of  $u$  and  $v$  there exist non-commuting elements  $a, b, c \in G$  such that  $a^2 b^2 c^r = 1$ .*

*Proof.* The one-relator group  $H = \langle a, b, c \mid a^2 b^2 c^r = 1 \rangle$  can also be written in terms of the presentation  $\langle b, c, d \mid b^{-1} d^{-1} b = c^{-r} d \rangle$ . This can be seen by letting  $d = ab$  and writing the relation  $a^2 b^2 c^r = 1$  as  $db^{-1} db^{-1} b^2 c^r = 1$ , which can then be rewritten as  $b^{-1} d^{-1} b = c^{-r} d$ .

Thus in the HNN extension  $\langle b, c, d \mid b^{-1}d^{-1}b = c^{-r}d \rangle$  of the free group generated by  $\{c, d\}$ , with stable letter  $b$  and associated subgroups  $\langle d \rangle$  and  $\langle c^{-r}d \rangle$ , the equality  $a^2b^2c^r = 1$ , where  $a = db^{-1}$ , will be satisfied, but none of  $a, b, c$  will commute.

We can clearly take the HNN extension of any finitely generated non-cyclic free group  $F$  with associated cyclic subgroups of the form  $\langle x \rangle$  and  $\langle y^r x \rangle$ , where  $x$  and  $y$  are generators of  $F$ , and an equality of the form  $a^2b^2c^r = 1$  will be satisfied without any of  $a, b, c$  commuting.  $\square$

For brevity we refer the reader to [2] for a complete account of  $\Lambda$ -trees and the groups that act freely on them, called  $\Lambda$ -free groups.

**Corollary 1.** *There exist  $\Lambda$ -free groups in which*

$$a^2b^2c^r = 1$$

*holds for non-commuting  $a, b, c \in G$ , and  $r \geq 2$ .*

*Proof.* In [10, Theorem 3.1] it is shown that groups with a presentation of the form  $\langle x, y, x_1, \dots, x_n \mid xyx^{-1}y^e = w \rangle$ , where  $w$  is any word in  $\{x_1, \dots, x_n\}$ , act freely on  $(\mathbb{Z} \times \mathbb{Z})$ -trees, unless  $e = 1$  and  $w = 1$ . The groups arising in the proof of Theorem 1, i.e. the groups  $\langle b, c, d \mid b^{-1}d^{-1}b = c^{-r}d \rangle$  where  $r \geq 2$ , have this form and will therefore act on a  $\Lambda$ -tree. They also satisfy the equation  $a^2b^2c^r = 1$ , where  $a = db^{-1}$ , with non-commuting  $a, b, c$ .  $\square$

Before we state the next results we need to make the following observations. Let  $F$  be a free group with basis  $X$ . We remind the reader (see for example [8, Chapter I.4]) that a *Whitehead automorphism* of  $F$  is an automorphism  $\tau$  of one of the following two kinds:

- (1)  $\tau$  permutes the elements of  $X^{\pm 1} = X \cup X^{-1}$ ;
- (2) for some fixed  $a \in X^{\pm 1}$ ,  $\tau$  carries each of the elements  $x \in X^{\pm 1}$  into one of  $x, a^{-1}x, xa$  or  $a^{-1}xa$ .

We now consider the special situation with  $X = \{x_1, x_2, x_3\}$  and  $F$  free on  $X$ . In  $F$  we consider the two words

- (1)  $w_1(x_1, x_2, x_3) = x_1^p x_2^q x_3^r$  with  $p \geq 2, q \geq 3, r \geq 3$ ;
- (2)  $w_2(x_1, x_2, x_3) = g_1 x_3 u^{\alpha_1} x_3^{-1} g_2 x_3 u^{\alpha_2} x_3^{-1} \dots g_n x_3 u^{\alpha_n} x_3^{-1}$  with  $n \geq 1, 1 \neq u = u(x_1, x_2)$  and  $u$  is not conjugate to a power of  $x_1$  or  $x_2$ ,  $\alpha_i$  non-zero integers and  $g_1, \dots, g_n$  freely reduced words in  $\langle x_1, x_2 \rangle$ , with  $g_i \neq 1$  for  $i = 1, \dots, n$ .

**Remark 1.** Assume that  $p, q, r \geq 3$ . We first note that  $w_1 \neq w_2$  and  $w_1$  is *minimal* (with respect to length) in its automorphic orbit. It can be easily seen that if  $\tau$  is a Whitehead automorphism of  $F$  of type (2), if we apply  $\tau$  to  $w_1(x_1, x_2, x_3)$ , then the length strictly increases. The minimality of  $w_1$  also shows that it cannot be a primitive

element by [8, Proposition 4.17]. In fact,  $w_1$  is a word of minimal rank, also called a *regular* word, that is, there is no Nielsen transformation from  $\{x_1, x_2, x_3\}$  to a system  $d, f, g$  with  $x_1^p x_2^q x_3^r \in \{d, f\}$  (see [6]).

**Remark 2.** We now consider  $w_2(x_1, x_2, x_3)$ . If  $w_2$  is minimal, then there is no Whitehead automorphism  $\tau$  such that the length strictly decreases when applying  $\tau$ . Hence, if  $w_2$  is minimal, there is no automorphism taking  $w_1$  to  $w_2$  by [8, Proposition 4.17], as the only automorphism taking  $w_1$  to  $w_2$  would be a permutation, and the form of the two words does not allow for a permutation to send  $w_1$  to  $w_2$ . If  $w_2$  is not minimal, then each Whitehead automorphism which decreases the length of  $w_2$  will take  $w_2$  to a word of the same form. To see this, notice that  $u$  contains both  $x_1$  and  $x_2$ . If, for instance,  $g_1 = g'_1 x_2$ , then an automorphism which replaces  $x_2$  by  $x_2 x_3 = x'_2$  gives  $x'_2 x_3^{-1}$  at all other places where  $x_2$  occurred, especially inside  $u$ .

**Remark 3.** If  $p = 2$  and  $q, r \geq 3$ , then  $w_1$  is still minimal, and when we apply a Whitehead automorphism  $\tau$  to  $w_1(x_1, x_2, x_3)$  the length strictly increases, except when  $\tau$  is of the form  $x_1 \rightarrow x_1 x_2^{-1}, x_2 \rightarrow x_2, x_3 \rightarrow x_3$ , in which case the length stays the same. If  $w_2$  is minimal, the only automorphisms that could take  $w_1$  to  $w_2$  are of the form  $\tau$  composed with permutations, and one can see that such automorphisms cannot take  $w_1$  to a word of the form  $w_2$ . As in Remark 2, if  $w_2$  is not minimal, then each Whitehead automorphism which decreases the length of  $w_2$  will take  $w_2$  to a word of the same form.

From the minimality of  $w_1$  and the facts about  $w_2$  in the above paragraphs we get the following.

**Lemma 1.** *Let  $F$  be free with basis  $\{x_1, x_2, x_3\}$  and  $w = w_1(x_1, x_2, x_3) = x_1^p x_2^q x_3^r$  with  $p \geq 2, q \geq 3, r \geq 3$ . Then there is no automorphism  $\alpha$  of  $F$  with  $\alpha(x_i) = y_i, i = 1, 2, 3$ , such that  $\alpha^{-1}(w)$  is, written in  $y_1, y_2, y_3$ , of the form*

$$g_1 y_3 u^{\alpha_1} y_3^{-1} g_2 y_3 u^{\alpha_2} y_3^{-1} \dots g_n y_3 u^{\alpha_n} y_3^{-1}$$

with  $n \geq 1, 1 \neq u = u(y_1, y_2)$  and  $u$  not conjugate to a power of  $y_1$  or  $y_2$ , all  $\alpha_i$  non-zero integers, and  $g_1, \dots, g_n$  freely reduced words in  $\langle y_1, y_2 \rangle$ , with  $g_i \neq 1$  for  $i = 1, \dots, n$ .

Lemma 1 states that words of the form  $w_1$  and  $w_2$  cannot be in the same automorphic orbit.

**Theorem 2.** *Let  $F$  be a finitely generated free group,  $u$  and  $v$  non-trivial elements in  $F$  that are not proper powers, and  $G = \langle F, t \mid tut^{-1} = v \rangle$ . Let  $a, b, c \in G$  satisfy  $a^p b^q c^r = 1$  with  $p \geq 2, q \geq 3, r \geq 3$ .*

- (i) *If  $u$  is not conjugate to  $v^{-1}$ , then  $a, b, c$  must commute.*
- (ii) *If  $u$  is conjugate to  $v^{-1}$ , then  $a, b, c$  either commute or generate the Klein bottle group  $\langle x, y \mid xyx^{-1}y = 1 \rangle$ .*

*Proof.* Let  $H = \langle a, b, c \rangle$ . We will consider three cases:

- (1)  $u$  and  $v^{\pm 1}$  are not conjugate;
- (2)  $u$  and  $v$  are conjugate;
- (3)  $u$  and  $v^{-1}$  are conjugate.

In case (1) assume that  $H$  is not abelian. Then  $H$  cannot be free of rank 2 or 3 because the word  $a^p b^q c^r$  is regular by Remark 1. Thus, by [4, Theorem 2.2],  $H$  must be a one-relator group, that is, in our case,  $H = \langle a, b, c \mid a^p b^q c^r = 1 \rangle$ . Furthermore, by the proof of [4, Theorem 2.2], there is a Nielsen transformation from  $\{a, b, c\}$  to a system  $\{x_1, x_2, x_3\}$  for which we may assume, without loss of generality, that  $x_1, x_2 \in F$ ,  $\langle x_1, x_2 \rangle$  non-cyclic,  $x_3 = t$  and  $H$  has a presentation of the form

$$H = \langle x_1, x_2, x_3 \mid w(x_1, x_2, x_3) = 1 \rangle,$$

with  $w(x_1, x_2, x_3) = g_1 x_3 h^{\alpha_1} x_3^{-1} g_2 x_3 h^{\alpha_2} \dots g_n x_3 h^{\alpha_n} x_3^{-1}$ ,  $n \geq 1$ ,  $h = u^\alpha \in \langle x_1, x_2 \rangle$ ,  $\alpha \neq 0$ ,  $\alpha_i \neq 0$  and  $g_i \in \langle x_1, x_2 \rangle$  non-trivial and freely reduced for  $i = 1, \dots, n$ . But this contradicts Lemma 1 if  $h$  is not conjugate to a power of  $x_1$  or  $x_2$ . Therefore  $H$  is abelian if  $h$  is not conjugate to a power of  $x_1$  or  $x_2$ .

Now suppose that  $h$  is conjugate to a power of  $x_1$  or  $x_2$ . Without loss of generality we may assume that  $h = x_1^\gamma$ . Since  $h = u^\alpha$  and  $u$  is not a proper power in  $F$  we get that  $x_1 = u^\delta$  in  $F$ . With respect to the equation  $a^p b^q c^r = 1$  and because  $H = \langle a, b, c \rangle$  we may replace  $x_1$  by  $u$ . Hence, let  $x_1 = u$ . We may also assume that  $x_2$  is not a proper power in  $F$ . Now let both  $x_1$  and  $x_2$  be not a proper power in  $F$ . Using this and the cancellation arguments in [4, Theorems 1 and 2], we see that  $v$  is in  $\langle x_1, x_2 \rangle$  because  $H$  is not free of rank 2 or 3. Hence,  $H$  has a presentation of the form  $K = \langle x_1, x_2, t \mid tx_1 t^{-1} = v \rangle$  with  $v = v(x_1, x_2)$  freely reduced in  $x_1$  and  $x_2$ , and  $t = x_3$ . But in the free group on  $a, b, c$  there is no automorphism  $\varphi$  with  $\varphi(a) = x_1$ ,  $\varphi(b) = x_2$  and  $\varphi(c) = t$  such that  $\varphi^{-1}(a^p b^q c^r)$  with  $2 \leq p, 3 \leq q, 3 \leq r$  is, written in  $x_1, x_2$  and  $t$ , of the form  $tx_1 t^{-1} v^{-1}$ . This gives a contradiction. Hence,  $H$  is abelian in this case as well.

In case (2), we can assume without loss of generality that  $u = v$ . Then  $G$  is fully residually free, which implies that  $H$  is fully residually free. Thus  $H$  has the same universal theory as that of free groups. Therefore  $a^p b^q c^r = 1$  with  $p, q, r \geq 2$  implies that  $a, b$  and  $c$  commute.

In addition we remark that, for the case  $u = v$ , by the classification given in [7, Theorem 5], any non-abelian, non-free rank 3 subgroup  $K$  of  $G$  is a free rank one extension of centralizers of a free group of rank 2, that is, in our case:  $K = \langle x_1, x_2, x_3 \mid x_3 h x_3^{-1} = h \rangle$  with  $h \in \langle x_1, x_2 \rangle$ ; additionally, either  $h$  is regular and not a proper power in  $G$  or  $h$  is not regular, in which case  $K$  is isomorphic to  $\langle x, y, \mid xy = yx \rangle \star \langle z \mid \rangle$  (the free product of a free abelian group of rank 2 and the integers).

In case (3), we can assume without loss of generality that  $u = v^{-1}$ , that is,  $G = \langle F, t \mid tut^{-1} = u^{-1} \rangle$ . Since  $t^2 u t^{-2} = u$ , one can easily extend the arguments in [7, Theorem 5] (which only rely on the Nielsen cancellation method, and no residual

properties, in a group with relation  $u = v$ , in order to obtain a classification of rank 3 subgroups), regarding non-abelian, non-free rank 3 subgroups of fully residually free groups to the case  $u = v^{-1}$  and obtain that any non-abelian, non-free rank 3 subgroup  $K$  of  $G$  has a presentation  $K = \langle x_1, x_2, x_3 \mid x_3 h x_3^{-1} = h^\varepsilon \rangle$ , with  $\varepsilon = \pm 1$  and  $h \in \langle x_1, x_2 \rangle$ ; additionally, either  $h$  is regular and not a proper power in  $G$  or  $h$  is not regular, in which case  $K$  is isomorphic to  $\langle x, y \mid x y x^{-1} = y^\varepsilon \rangle \star \langle z \mid \rangle$ , where  $\varepsilon = 1$  or  $\varepsilon = -1$ .

We assume now that  $H = \langle a, b, c \rangle$  is not free, not abelian and of rank 3. Then  $H$  is isomorphic to a subgroup  $K$  of  $G$  as described above, that is,  $H \cong K = \langle x_1, x_2, x_3 \mid x_3 h x_3^{-1} = h^\varepsilon \rangle$ . If  $\varepsilon = 1$ , then  $H$  is fully residually free and so  $a^p b^q c^r = 1$  implies that  $a, b$  and  $c$  commute.

Now let  $\varepsilon = -1$ . In case  $h$  is regular, and not a proper power, then, if  $H$  is non-free and not abelian, by the above arguments  $H$  has to be a one-relator group, that is, in our case,  $H = \langle a, b, c \mid a^p b^q c^r = 1 \rangle$  since  $a^p b^q c^r$  is regular. By the same arguments as in case (1), there must be a Nielsen transformation from  $\{a, b, c\}$  to a system  $\{x_1, x_2, x_3\}$  for which, without loss of generality,  $H$  has a presentation of the form  $\langle x_1, x_2, x_3 \mid x_3 h x_3^{-1} h = 1 \rangle$ . But  $x_3 h x_3^{-1} h$  is a word of the form  $w_2$ , and so by Lemma 1 this cannot happen.

If  $h$  is not regular, then  $H \cong K = \langle x, y \mid x y x^{-1} = y^{-1} \rangle \star \langle z \mid \rangle$ . But since  $a, b, c$  satisfy  $a^p b^q c^r = 1$  and  $a^p b^q c^r$  is regular, [6, Theorem 5.2] implies that  $H$  must in fact be a rank 2 subgroup, which is not the case.

Finally, let  $H = \langle a, b, c \rangle$  be a non-abelian, non-free rank 2 subgroup. Then by [3, Theorem 1]  $H$  must be the Klein bottle group

$$V = \langle x, y \mid x y x^{-1} = y^{-1} \rangle \cong \langle u, v \mid u^2 = v^2 \rangle.$$

In  $V$  the elements  $u$  and  $v$  do not commute. Thus by taking, for instance,  $a = u$ ,  $b = u^{-1}$  and  $c = v^{-1}$ , since  $u^{12} u^{-8} = v^4$ , one gets that  $a^{12} b^8 c^4 = 1$ .  $\square$

The groups in Theorem 2 for which the translation lengths of  $u$  and  $v$  are equal are in fact  $\Lambda$ -free groups by Bass' work (see [10, Theorem 2.4.1]). We may extend Theorem 2 to the following result, after reminding the reader that a group  $G$  is called  $n$ -free for a positive integer  $n$  if every subgroup of  $G$  generated by  $n$  elements is free.

**Theorem 3.** *Let  $L$  be a non-cyclic, 2-free, fully residually free group,  $u$  and  $v$  non-trivial elements in  $L$  that are not proper powers and  $u$  is not conjugate to  $v^{-1}$ . Let  $G = \langle L, t \mid t u t^{-1} = v \rangle$ . If  $a, b, c \in G$  satisfy  $a^p b^q c^r = 1$  for  $p \geq 2, q \geq 3, r \geq 3$  then  $a, b, c$  must commute.*

*Proof.* We first remark that  $L$  is also 3-free by [7].

If  $u$  is conjugate to  $v$ , then we may assume that  $u = v$ . Then  $G$  is fully residually free and hence Theorem 3 holds.

From now on we assume that  $u$  is not conjugate to  $v$ . In the proof of Theorem 2 we used the classification of the rank 3 subgroups of  $G$  for the case that  $L$  is a non-

abelian free group (see [4]). In [4], in addition to the standard Nielsen cancellation method in HNN groups, one only needs three properties of  $L$  and  $G$  respectively:

- (1) the subgroups  $\langle u \rangle$  and  $\langle v \rangle$  are malnormal in  $L$ ;
- (2)  $L$  is 3-free;
- (3) each two-generator subgroup of  $G$  is free.

Now let  $L$ , as in the statement of Theorem 3, be a non-cyclic, 2-free, fully residually free group. We have to show that the properties (1), (2) and (3) also hold in this more general situation.

(1) holds because  $L$  is 2-free. Let  $x \in L$  be such that  $xu^\alpha x^{-1} = u^\beta$  for some integers  $\alpha, \beta \neq 0$ . Since  $L$  is 2-free, the subgroup  $\langle x, u \rangle$  of  $L$  is cyclic. Hence  $x \in \langle u \rangle$ .

(2) holds by the above remark that  $L$  is 3-free.

We now show that (3) also holds. In [3], in the special case that  $L$  is a non-abelian free group we have used, besides the Nielsen cancellation method in HNN groups and property (1), only the fact that  $L$  is 2-free. But this we assume anyway for  $L$ . Hence (3) also holds for the more general situation. We may now apply analogous arguments to the ones in the proof of Theorem 2.  $\square$

**Corollary 2.** *Let  $S = \langle a_1, b_1, \dots, a_n, b_n \mid [a_1, b_1] \dots [a_n, b_n] = 1 \rangle$ ,  $n \geq 2$ , be an orientable surface group of genus  $\geq 2$  or  $S = \langle a_1, \dots, a_n \mid a_1^2 \dots a_n^2 = 1 \rangle$  a non-orientable surface group of genus  $\geq 4$ . Let  $u$  and  $v$  be non-trivial elements in  $S$  that are not proper powers and  $u$  is not conjugate to  $v^{-1}$ , and let  $G = \langle S, t \mid tut^{-1} = v \rangle$ . Then if for  $a, b, c \in G$  and  $p \geq 2$ ,  $q \geq 3$ ,  $r \geq 3$  the equality  $a^p b^q c^r = 1$  holds, the elements  $a, b, c$  must commute.*

*Proof.* In both cases  $S$  is a non-cyclic, 2-free, fully residually free group. Hence Corollary 2 holds by Theorem 3.  $\square$

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