1-1-2009

Extraction of Complex Permittivity of Multilayered Dielectric Sample Loaded in a Rectangular Waveguide

Uma Balaji
Fairfield University, ubalaji@fairfield.edu

Extraction of Complex Permittivity of Multilayered Dielectric Sample Loaded in a Rectangular Waveguide

Uma Balaji
Electrical & Computer Engineering Department, California State University
Chico, CA 95929, USA

Abstract— Complex permittivity of dielectric materials loaded in the cross section of a rectangular waveguide can be determined from the S-parameter measurements of the discontinuity. The analytical electromagnetic tool used to solve the scattering problem of dielectric loaded waveguide is mode matching method. Minimization of an error function to extract the unknown complex permittivity of the material is done using a practical Quasi-Newton algorithm.

1. INTRODUCTION

The determination of permittivity of a dielectric material at high frequencies using transmission line technique is very popular. At frequencies starting from few GHz, waveguides are used as the transmission line and the dielectric sample is placed such that it completely or partially fills the cross-section of the waveguide. The S-parameter of the discontinuity is measured and the complex permittivity is extracted from this measurement by solving the inverse problem [1–3]. An error function that compares the measured and calculated S-parameters of the discontinuity is minimized in order to extract the complex permittivity. The theoretically calculated S-parameters of such a discontinuity are usually based on methods such as finite element method, moment method and mode matching method. In this paper the multilayered dielectric sample is placed in the cross section of the waveguide to form a series of discontinuities and mode matching method has been used to calculate the S-parameter of such series of discontinuities. The determination of the complex permittivity of the material filling the waveguide from the measured S-parameters is an inverse problem which is solved using a suitable optimization technique. The optimization methods are often based on genetic algorithms [1]. The optimization algorithm used in this paper is based on the practical Quasi-Newton algorithm as described in [4]. The error function that has been minimized is as given in [1].

2. THEORY

The method to calculate the theoretical value of S-parameter of the discontinuity from an empty waveguide to a single layer of dielectric filled waveguide as in Fig. 1 using mode matching method is discussed first. The analysis of such a discontinuity using mode matching method involves the following steps. The fields on both sides of the discontinuity are expanded in terms of a series of modes of incident and reflected waves. The magnitude of power carried by each of the modes is set to unity. The continuity conditions for the tangential components of electric and magnetic fields are imposed. Using the principle of orthogonality of modes, the equations of continuity conditions are transformed into matrices relating the expansion coefficients of incident and reflected waves at the discontinuity. The matrices are rearranged and inverted suitably to obtain the generalized scattering matrix which describes the discontinuity in terms of the dominant and higher order modes. Theoretically the generalized scattering matrix is of infinite dimension corresponding to the infinite number of modes. The matrix is truncated to a finite size for numerical computations. However for a discontinuity as in Fig. 1, it is sufficient to match only the fields of the dominant mode alone in order to obtain the S-parameters as the higher order modes do not couple with the fundamental mode. The computation of S-parameters from this method is given in the appendix.

In order to determine the S-parameters of multilayered dielectric sample, the following procedure is adopted. The discontinuity at each interface between two consecutive layers of the dielectric sample is analyzed using the mode matching method independently. While analyzing the discontinuity at the interface of two consecutive layers it is assumed that the material is ideal and hence the permittivity is real. However since each layer is of finite length, the S-parameter of the discontinuity from one layer to another layer of finite length is calculated considering the loss tangent and relative permittivity of that layer. This leads to the S-parameters of discontinuities at the interface of all the layers of multilayered dielectric sample including the losses that take place in all the layers of dielectric loaded waveguide. In order to incorporate the fact that the final layer of the

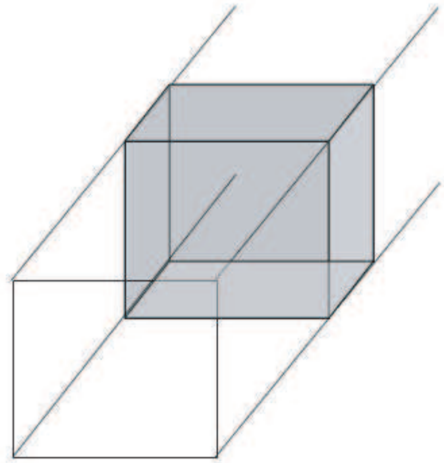

Figure 1: Dielectric sample in cross-section of waveguide.

sample ends in an empty section of waveguide, the S-parameters of a discontinuity from the final layer of dielectric sample to an empty waveguide is calculated. These S-parameters are all cascaded according to the placement of each layer to obtain the theoretical S-parameter of the multilayered discontinuity.

The inverse problem of extraction of complex permittivity is based on an initial guess for its value in the above algorithm that evaluates S-parameters of the discontinuity during optimization process. An error function that compares the theoretically computed S-parameter to the measured S-parameter is minimized during the optimization. The evaluation of S-parameter of the discontinuity is done over a narrow range of frequencies.

The S-parameter of such a structure with multilayered dielectric sample of unknown complex permittivity placed in the waveguide section is measured over the desired narrow range of frequencies. The error function E that is minimized by the optimization algorithm is calculated at these N discrete frequency points and is given by the function below.

$$E = \frac{1}{N}\sum_{i=1}^{N}\left\{ \begin{array}{l} (\mathrm{Re}[S_{11}]_{c,i} - \mathrm{Re}[S_{11}]_{m,i})^2 + (\mathrm{Re}[S_{12}]_{c,i} - \mathrm{Re}[S_{12}]_{m,i})^2 \\ +(\mathrm{Im}[S_{11}]_{c,i} - \mathrm{Im}[S_{11}]_{m,i})^2 + (\mathrm{Im}[S_{12}]_{c,i} - \mathrm{Im}[S_{12}]_{m,i})^2 \end{array} \right\}$$

The subscript c in the S-parameters indicates calculated values and the subscript m denotes the measured values. Minimization of the error function is performed using a practical Quasi-Newton algorithm [4].

3. RESULTS

A program to analyze the discontinuity of dielectric loaded waveguide was developed and tested based on the unitary property of the S-matrix when the losses in the network are zero. In order to do this the loss tangent of the dielectric sample was set to zero. The optimization algorithm was verified by finding the minimum of the Rosenbrock function. The minimum of this function which occurs at [1 1] was evaluated in 32 iteration by the program when a choice of initial guess was $[-1\ \ -1]$.

The S-parameter of finite length of 2 mm discontinuity with teflon as dielectric sample of relative permittivity 2.2 and loss tangent 0.0009 in the WR 90 waveguide as in Fig. 1 was evaluated at eleven frequency points between 10–10.1 GHz and set as the measured value. The optimization algorithm was run with an initial guess of relative permittivity of 1.1 and loss tangent 1e-6. A relative permittivity of 2.2 and loss tangent of 0.0009 was obtained in 15 iterations. This is shown in Fig. 2 and Fig. 3.

The S-parameter of two layers of dielectric sample in the WR 90 waveguide of relative permittivity 2.2 and loss tangent 0.0009 followed by relative permittivity 3.5 and loss tangent 0.0001 was evaluated at eleven frequency points between 10–10.1 GHz and set as the measured value. The optimization algorithm was run with an initial guess of relative permittivity of 1.1 and loss tangent 9e-5 for the first layer and 1.1 and 1e-5 for the second layer. Convergence to the exact value was obtained in 25 iterations. This is shown in Fig. 4 and Fig. 5.

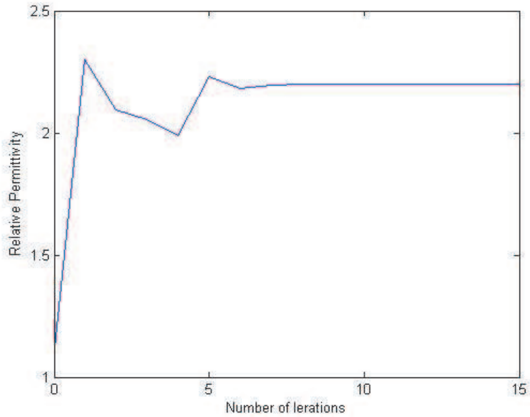

Figure 2: Convergence of relative permittivity of dielectric sample in Fig. 1.

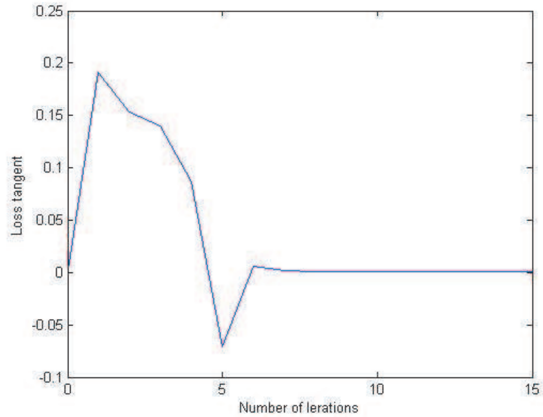

Figure 3: Convergence of loss tangent of dielectric sample in Fig. 1.

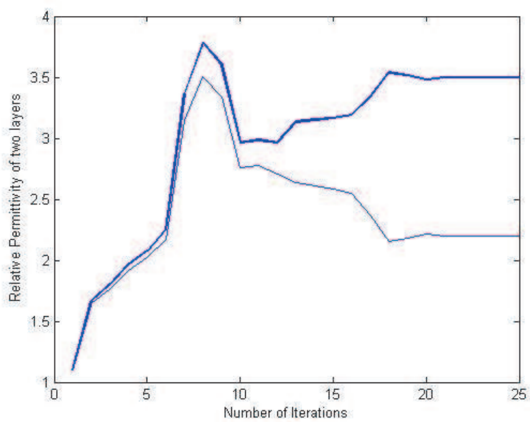

Figure 4: Convergence of relative permittivity for a two layered dielectric sample loaded in a rectangular waveguide.

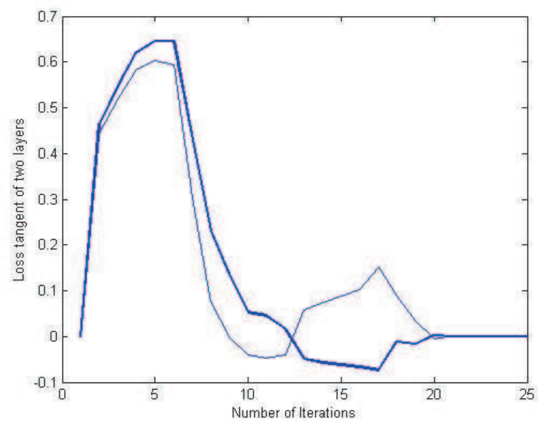

Figure 5: Convergence of loss tangent for a two layered dielectric sample loaded in a rectangular waveguide.

4. CONCLUSION

This paper has described a technique to determine complex permittivity of multilayered dielectric material. The optimization algorithm used in this paper has been found to give excellent results with the error function that has been used.

Appendix

The determination of S-matrix of discontinuity from empty waveguide to dielectric filled waveguide is given below.

$$L = \sqrt{\frac{k_{zm}^{I}}{k_{zi}^{II}}}$$

In above, k_{zm}^{I} is phase constant of fundamental mode in the empty waveguide and k_{zi}^{II} is the phase constants of the fundamental mode in dielectric filled waveguide.

The S-parameters of the discontinuity is obtained from

$$S_{11} = (LL+1)^{-1}(LL-1); \qquad S_{12} = 2(LL+1)L$$
$$S_{21} = L(1-S_{11}); \qquad S_{22} = 1 - LS_{12}$$

For determining the scattering matrix of the discontinuity of finite length, say l, of the dielectric sample with complex propagation constant γ of the fundamental mode, in an empty waveguide is determined from the following equations:

$$\begin{aligned} R &= [1 - S_{22}DS_{22}D]^{-1} \\ D &= e^{-\gamma z} \\ S_{011} &= S_{11} + S_{12}DRS_{22}DS_{21} = S_{022} \\ S_{012} &= S_{12}DRS_{21} = S_{021} \end{aligned}$$

REFERENCES

1. Faircloth, D. L., et al., "Complex constitutive parameter extraction for multilayered samples using S-parameter waveguide measurement," *Proceedings of IEEE Antennas and Propagation Symposium*, 4833–4835, 2006.
2. Barba, P., et al., "S-parameter extraction of off-set material samples in a waveguide," *Proceedings of IEEE Antennas and Propagation Symposium*, 4829–4832, 2006.
3. Balaji, U., "Complex permittivity extraction of dielectric samples in waveguide," *Proceedings of International Conference on Microwave and Millimeter Wave Technology, (ICMMT)*, Nanjing, China, April 21–24, 2008.
4. Antoniou, A., *Digital Filters, Design and Applications*, Second edition, McGraw-Hill, 1993.