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Simultaneous target state and passive sensors bias estimation

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***Abstract*—Most of the literature pertaining to target tracking assumes that the sensor data are corrupted by measurement noises that are zero mean (i.e., unbiased) and with known variances (accuracies). However in real tracking systems, measurements from sensors exhibit, typically, biases. For angle-only sensors, imperfect registration leads to Line Of Sight (LOS) measurement biases in azimuth and elevation. In this project we propose a new methodology that uses an exoatmospheric target of opportunity seen in a satellites borne sensor's field of view to estimate the sensor's biases simultaneously with the state of the target. The first step is to formulate a general bias model for synchronized optical sensors; then we use a Maximum Likelihood (ML) approach that leads to a nonlinear least-squares estimation problem for simultaneous estimation of the 3D Cartesian position and velocity components of the target of opportunity and the angle measurement biases of the sensors (two in the present study). Each satellite is equipped with an IR sensor that provides LOS measurements (azimuth and elevation) to the target. The measurements provided by these sensors are assumed to be noisy and biased but perfectly associated, i.e., it is known perfectly that they belong to the same target. The sensor bias and the target state estimates, obtained via Iterative Least Squares (ILS), are shown, by the simulation, to be unbiased.**

***Index Terms*—Bias estimation, space tracking, observability, composite measurements, maximum likelihood.**

I. Introduction

A single-target tracking IR system will, typically, attempt to keep the target centered in the sensor field of view and provide measurements of target LOS angles to an algorithm that estimates a target state such as position, velocity, and acceleration. Methods to enhance sensor LOS accuracy and resolution of target state dynamics will minimize track uncertainties and enhance track state estimation. A space-based tracking system provides many advantages for missile defense as well as space situational awareness as a part of a system of systems that contribute to an overall picture. It can cover gaps in terrestrial radar coverage and expand the capabilities of a Ballistic Missile Defense System (BMDS), allow interceptors to engage enemy missiles earlier in their trajectories, discriminate between warheads and decoys, and provide warhead hit assessment. However, systemic errors in sensing systems hinder accurate threat identification and target state estimation, and, in this way, the space-based tracking systems present some unique challenges [6].

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Multisensor systems use fusion of data from multiple sensors to form accurate estimates of a target track. To fuse multiple sensor data the individual sensor data must be expressed in a common reference frame. A problem encountered in multisensor systems is the presence of errors due to sensor bias. Bias error in a spacecraft and sensors can result from a number of different sources, including: errors in spacecraft position (spacecraft navigation bias); errors in spacecraft attitude (wheel assembly controller error, coordinate system translation round-off error); errors in sensor calibration (residual pointing error, degradation of sensor alignment); and errors in timing caused by bias in the clocks of the sensors. In [7] time varying bias estimation based on a nonlinear least squares formulation and the singular value decomposition using truth data was presented. An approach using maximum a posteriori (MAP) data association for concurrent bias estimation and data association based on sensor-level track state estimates was proposed in [8] and extended in [9].

For angle-only sensors, imperfect registration leads to LOS angle measurement biases in azimuth and elevation. If not corrected, the registration errors can seriously degrade the global surveillance system performance by increasing the tracking errors and even introducing ghost targets. In [5] the effect of sensor and timing bias error on the tracking quality of a space-based infrared (IR) tracking system that utilizes a Linearized Kalman Filter (LKF) for the highly non-linear problem of tracking a ballistic missile was presented. This was extended in [6] by proposing a method of using stars observed in the sensor background to reduce the sensor bias error. In [3] simultaneous sensors bias and targets position estimation using fixed passive sensors was proposed. A solution to the related observability issues discussed in [3] is proposed in [4] using space based sensors.

The new bias estimation algorithm developed in this paper, is validated using a hypothetical scenario created using System Tool Kit (STK) [1]. The tracking system consists of two optical sensors (space based) tracking a ballistic target. We assume the sensors are synchronized, their locations are known, and the data association is correct; and we estimate their orientation biases while simultaneously estimating the state of the target (position and velocity).

Section II presents the problem formulation and solution in detail. Section III describes the simulations performed and gives the results. Finally, Section IV gives the conclusions and future work.

II. Problem Formulation

An important prerequisite for successful multisensor integration (fusion) is that the data from the reporting sensors are transformed to a common reference frame free of systematic or registration errors (biases). The fundamental frame of reference used in this paper is the Earth Centered Inertial ECI Coordinate System.

In a multisensor scenario, sensor platform s has a sensor reference frame associated with it (measurement frame of the sensor) defined by the orthogonal set of unit vectors $(e_{\xi_s}, e_{\eta_s}, e_{\zeta_s})$. The origin of the measurement frame of the sensor is a translation of the ECI origin, and its axes are rotated with respect to the ECI axes. The rotation between these frames can be described by a set of Euler angles. We will refer to these angles $\phi_s + \phi_s^{\mathrm{n}}$, $\rho_s + \rho_s^{\mathrm{n}}$, $\psi_s + \psi_s^{\mathrm{n}}$ of sensor s, as roll, pitch and yaw respectively, where ϕ_s^{n} is the nominal roll angle, ϕ_s is the roll bias, etc.

Each angle defines a rotation about a prescribed axis, in order to align the sensor frame axes with the ECI axes. The xyz rotation sequence is chosen, which is accomplished by first rotating about the x axis by ϕ_s^{n}, then rotating about the y axis by ρ_s^{n}, and finally rotating about the z axis by ψ_s^{n}. The rotations sequence can be expressed by the matrices

$$\begin{aligned} T_s(\psi_s^{\mathrm{n}}, \rho_s^{\mathrm{n}}, \phi_s^{\mathrm{n}}) =& T_z(\psi_s^{\mathrm{n}}) T_y(\rho_s^{\mathrm{n}}) T_x(\phi_s^{\mathrm{n}}) \\ =& \begin{bmatrix} \cos\psi_s^{\mathrm{n}} & \sin\psi_s^{\mathrm{n}} & 0 \\ -\sin\psi_s^{\mathrm{n}} & \cos\psi_s^{\mathrm{n}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ &\cdot \begin{bmatrix} \cos\rho_s^{\mathrm{n}} & 0 & -\sin\rho_s^{\mathrm{n}} \\ 0 & 1 & 0 \\ \sin\rho_s^{\mathrm{n}} & 0 & \cos\rho_s^{\mathrm{n}} \end{bmatrix} \\ &\cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi_s^{\mathrm{n}} & \sin\phi_s^{\mathrm{n}} \\ 0 & -\sin\phi_s^{\mathrm{n}} & \cos\phi_s^{\mathrm{n}} \end{bmatrix} \end{aligned} \quad (1)$$

Assume there are N_S synchronized passive sensors, with known positions in ECI coordinates,
$\boldsymbol{\xi}_s(k) = [\xi_s(k), \eta_s(k), \zeta_s(k)]' \quad s = 1, 2, ..., N_S$,
$k = 0, 1, 2, ..., K$, tracking a single target at unknown positions $\mathbf{x}(k) = [x(k), y(k), z(k)]'$, also in ECI coordinates.

With the previous convention, the operations needed to transform the position of the target location expressed in ECI coordinates into the sensor s coordinate system (based on its nominal orientation) is

$$\begin{aligned} \mathbf{x}_s^{\mathrm{n}}(k) = T(\boldsymbol{\omega}_s(k))(\mathbf{x}(k) - \boldsymbol{\xi}_s(k)) \\ s = 1, 2, ..., N_S, \quad k = 0, 1, 2, ..., K \end{aligned} \quad (2)$$

where $\boldsymbol{\omega}_s(k) = [\phi_s^{\mathrm{n}}(k), \rho_s^{\mathrm{n}}(k), \psi_s^{\mathrm{n}}(k)]'$ is the nominal orientation of sensor s, $T(\boldsymbol{\omega}_s(k))$ is the appropriate rotation matrix, and the translation $(\mathbf{x}(k) - \boldsymbol{\xi}_s(k))$ is the difference between the vector position of the target and the vector position of the sensor s, both expressed in ECI coordinates. The superscript "n" in (2) indicates that the rotation matrix is based on the nominal sensor orientation.

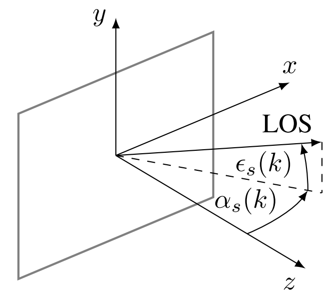

Fig. 1. Optical sensor coordinate system with the origin in the center of the focal plane.

Each passive sensor provides LOS measurements of the target position. As shown in Figure 1, the azimuth angle $\alpha_s(k)$ is the angle in the sensor xz plane between the sensor z axis and the line of sight to the target, while the elevation angle $\epsilon_s(k)$ is the angle between the line of sight to the target and its projection onto the xz plane, i.e.,

$$\begin{bmatrix} \alpha_s(k) \\ \epsilon_s(k) \end{bmatrix} = \begin{bmatrix} \tan^{-1}\left(\frac{x_s(k)}{z_s(k)}\right) \\ \tan^{-1}\left(\frac{y_s(k)}{\sqrt{x_s^2(k) + z_s^2(k)}}\right) \end{bmatrix} \quad (3)$$

The model for the biased noise-free LOS measurements is then

$$\begin{aligned} \begin{bmatrix} \alpha_s^b(k) \\ \epsilon_s^b(k) \end{bmatrix} &= \begin{bmatrix} h_1(\mathbf{x}(k), \boldsymbol{\xi}_s(k), \boldsymbol{\omega}_s(k), \mathbf{b}_s) \\ h_2(\mathbf{x}(k), \boldsymbol{\xi}_s(k), \boldsymbol{\omega}_s(k), \mathbf{b}_s) \end{bmatrix} \\ &\triangleq \boldsymbol{h}(\mathbf{x}(k), \boldsymbol{\xi}_s(k), \boldsymbol{\omega}_s(k), \mathbf{b}_s) \end{aligned} \quad (4)$$

where h_1 and h_2 denote the sensor Cartesian coordinates-to-azimuth/elevation angle mapping that can be found by inserting (2) and (3) into (4), and the bias vector of sensor s is

$$\mathbf{b}_s = [\phi_s, \rho_s, \psi_s]' \quad (5)$$

At time k, each sensor provides the noisy LOS measurements

$$\mathbf{z}_s(k) = \boldsymbol{h}(\mathbf{x}(k), \boldsymbol{\xi}_s(k), \boldsymbol{\omega}_s(k), \mathbf{b}_s) + \mathbf{w}_s(k) \quad (6)$$

Let $\mathbf{z}$ be an augmented vector consisting of the batch stacked measurements from all the sensors up to time K

$$\mathbf{z} = [z_1(1), z_2(1), ..., z_{N_S}(1), ..., z_1(K), z_2(K), ..., z_{N_S}(K)] \quad (7)$$

and

$$\mathbf{w}_s(k) = [w_s^{\alpha}(k), w_s^{\epsilon}(k)]' \quad (8)$$

The measurement noises $\mathbf{w}_s(k)$ are zero-mean, white Gaussian with

$$R_s = \begin{bmatrix} (\sigma_s^{\alpha})^2 & 0 \\ 0 & (\sigma_s^{\epsilon})^2 \end{bmatrix} \quad s = 1, 2, ..., N_S \quad (9)$$

and are assumed mutually independent. The problem is to estimate the bias vectors for all sensors and the state vector (position and velocity) of the target of opportunity

$$\boldsymbol{\theta} = [x(K), y(K), z(K), \dot{x}(K), \dot{y}(K), \dot{z}(K), \mathbf{b}_1', ..., \mathbf{b}_{N_S}']' \tag{10}$$

from

$$\mathbf{z} = \mathbf{h}(\boldsymbol{\theta}) + \mathbf{w} \tag{11}$$

where

$$\begin{aligned}\mathbf{h}(\boldsymbol{\theta}) =& [h_{11}(\boldsymbol{\theta})', h_{21}(\boldsymbol{\theta})', ..., h_{N_S 1}(\boldsymbol{\theta})', ..., \\ & h_{1K}(\boldsymbol{\theta})', h_{2K}(\boldsymbol{\theta})', ..., h_{N_S K}(\boldsymbol{\theta})']\end{aligned} \tag{12}$$

$$\begin{aligned}\mathbf{w} =& [\mathbf{w}_1(1)', \mathbf{w}_2(1)', ..., \mathbf{w}_{N_S}(1)', ..., \\ & \mathbf{w}_1(K)', \mathbf{w}_2(K)', ..., \mathbf{w}_{N_S}(K)']\end{aligned} \tag{13}$$

and the covariance of the stacked process noise (13) is the $(N_s K \times N_s K)$ block-diagonal matrix

$$R = \begin{bmatrix} R_1 & 0 & \cdots & 0 \\ 0 & R_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & R_{N_S} \end{bmatrix} \tag{14}$$

A. Space target dynamics

The state space model for a discrete-time stochastic system is of the general form

$$\mathbf{x}(k+1) = f[\mathbf{x}(k), \mathbf{u}(k), \mathbf{v}(k)] \quad k = 0, 1, 2, ..., K \tag{15}$$

Although the motion of ballistic missiles in orbit about the Earth is nonlinear, with small time steps ($\leq 10s$) we can approximate the motion model with a discrete-time linear dynamic equation

$$\mathbf{x}(k+1) = F\mathbf{x}(k) + G\mathbf{u}(k) + G\mathbf{v}(k) \tag{16}$$

where $\mathbf{x}(\mathbf{k})$ is the 6 dimensional state vector at time k denoted as

$$\mathbf{x}(k) = [x(k), y(k), z(k), \dot{x}(k), \dot{y}(k), \dot{z}(k)]', \quad k = 0, 1, 2, ..., K \tag{17}$$

F is the state transition matrix, u is a known input representing the gravitational effects acting on the target, and $\mathbf{v}$ is the process noise (white noise acceleration) with covariance Q. The state transition matrix for a target with acceleration due to gravity is

$$F = \begin{bmatrix} 1 & 0 & 0 & \Delta t & 0 & 0 \\ 0 & 1 & 0 & 0 & \Delta t & 0 \\ 0 & 0 & 1 & 0 & 0 & \Delta t \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{18}$$

and the known input gain matrix (multiplying the appropriate components of the gravity vector) is

$$G = \begin{bmatrix} \Delta t^2/2 & 0 & 0 \\ 0 & \Delta t^2/2 & 0 \\ 0 & 0 & \Delta t^2/2 \\ \Delta t & 0 & 0 \\ 0 & \Delta t & 0 \\ 0 & 0 & \Delta t \end{bmatrix} \tag{19}$$

where Δt is the sampling interval. The gravity term is given by

$$\mathbf{u} = g\frac{\mathbf{x}_p}{a(\mathbf{x}_p)} \tag{20}$$

where $\mathbf{x_p}$ is the position part of the state $\mathbf{x}$ in (15), $g = 9.8$ m/s^2, and

$$a = \sqrt{x(k)^2 + y(k)^2 + z(k)^2} \tag{21}$$

is the distance from the target to the origin of the coordinates system. For simplicity we assume g to be constant. The ratio $\mathbf{x}_p/a$ yields the components of the gravity. of the target and provides the scaling factor for the gravity term. The process noise $\mathbf{v}$ accounts for the inaccurate modeling of the true system dynamics and is added to the state to model possible missile accelerations due to maneuvers with a covariance matrix Q,

$$Q = \begin{bmatrix} \sigma_x^2 & 0 & 0 \\ 0 & \sigma_y^2 & 0 \\ 0 & 0 & \sigma_z^2 \end{bmatrix} \tag{22}$$

We shall obtain the maximum likelihood (ML) estimate of the augmented parameter vector (10) consisting of the (unknown) target position, velocity and sensor biases (under the assumption $Q = 0$), by maximizing the likelihood function (LF) of $\boldsymbol{\theta}$ based on $\mathbf{z}$

$$\Lambda(\boldsymbol{\theta}; \mathbf{z}) = p(\mathbf{z}|\boldsymbol{\theta}) \tag{23}$$

The ML estimate (MLE) is then

$$\hat{\boldsymbol{\theta}}(\mathbf{z})^{ML} = \arg\max_{\boldsymbol{\theta}} \Lambda(\boldsymbol{\theta}; \mathbf{z}) \tag{24}$$

In order to find the MLE, one has to solve a nonlinear least squares problem. This will be done using a numerical search via the Batch Iterated Least Squares (ILS) technique.

B. Requirements for Bias Estimability

The necessary and sufficient requirement of bias estimability is the invertibility of the Fisher Information matrix (FIM). In order to have parameter observability, the FIM must be invertible. If the FIM is not invertible (i.e., it is singular), then the CRLB (the inverse of the FIM) will not exist — the FIM will have one or more infinite eigenvalues, which means total uncertainty in a subspace of the parameter space, i.e., ambiguity [2].

For the example of bias estimability discussed in the sequel, to estimate the biases of 2 sensors (6 bias components) and 6 target state components (3 position and 3 velocity components), i.e., the search is in an 12-dimensional space. As stated previously, the FIM must be invertible, so the rank of the FIM has to be equal to the number of parameters to be estimated (6+6=12, in the above example). The full rank of the FIM is a necessary and sufficient condition for estimability. There exists, however, a subtle unobservability for this example that will necessitate the use of more measurements than the strict minimum number of measurements.

C. Iterated Least Squares for Maximization of the LF of $\boldsymbol{\theta}$

Given the estimate $\hat{\boldsymbol{\theta}}^j$ after j iterations, the batch ILS estimate after the $(j+1)$th iteration will be

$$\hat{\boldsymbol{\theta}}^{j+1} = \hat{\boldsymbol{\theta}}^j + \left[(H^j)'R^{-1}H^j\right]^{-1}(H^j)'R^{-1}[\mathbf{z} - \mathbf{h}(\hat{\boldsymbol{\theta}}^j)] \quad (25)$$

where

$$\begin{aligned}\mathbf{h}(\hat{\boldsymbol{\theta}}^j) =&[h_{11}(\hat{\boldsymbol{\theta}}^j)', h_{21}(\hat{\boldsymbol{\theta}}^j)', ..., h_{N_S1}(\hat{\boldsymbol{\theta}}^j)' \\ &, ..., h_{1K}(\hat{\boldsymbol{\theta}}^j)', h_{2K}(\hat{\boldsymbol{\theta}}^j)', ..., h_{N_SK}(\hat{\boldsymbol{\theta}}^j)']\end{aligned} \quad (26)$$

where

$$H^j = \left.\frac{\partial \mathbf{h}\left(\boldsymbol{\theta}^j\right)}{\partial \boldsymbol{\theta}}\right|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}^j} \quad (27)$$

is the Jacobian matrix of the vector consisting of the stacked measurement functions (26) w.r.t. (10) evaluated at the ILS estimate from the previous iteration j. In this case, the Jacobian matrix is, with the iteration index omitted for conciseness,

$$H = \left[\begin{array}{ccccccc} H_{11} & H_{21} & H_{N_S1} & \cdots & H_{1K} & H_{2K} & H_{N_SK} \end{array}\right]' \quad (28)$$

where

$$H_{sk} = \left[\begin{array}{cccccccccccc} \frac{h_{1s}(k)}{\partial x(k)} & \frac{h_{1s}(k)}{\partial y(k)} & \frac{h_{1s}(k)}{\partial z(k)} & \frac{h_{1s}(k)}{\partial \dot{x}(k)} & \frac{h_{1s}(k)}{\partial \dot{y}(k)} & \frac{h_{1s}(k)}{\partial \dot{z}(k)} & \frac{h_{1s}(k)}{\partial b_{\alpha_1}} & \frac{h_{1s}(k)}{\partial b_{\epsilon_1}} & \frac{h_{1s}(k)}{\partial b_{\rho_1}} & \cdots & \frac{h_{1s}(k)}{\partial b_{\alpha_{N_S}}} & \frac{h_{1s}(k)}{\partial b_{\epsilon_{N_S}}} & \frac{h_{1s}(k)}{\partial b_{\rho_{N_S}}} \\ \frac{h_{2s}(k)}{\partial x(k)} & \frac{h_{2s}(k)}{\partial y(k)} & \frac{h_{2s}(k)}{\partial z(k)} & \frac{h_{2s}(k)}{\partial \dot{x}(k)} & \frac{h_{2s}(k)}{\partial \dot{y}(k)} & \frac{h_{2s}(k)}{\partial \dot{z}(k)} & \frac{h_{2s}(k)}{\partial b_{\epsilon_1}} & \frac{h_{2s}(k)}{\partial b_{\epsilon_1}} & \frac{h_{2s}(k)}{\partial b_{\rho_1}} & \cdots & \frac{h_{2s}(k)}{\partial b_{\epsilon_{N_S}}} & \frac{h_{2s}(k)}{\partial b_{\epsilon_{N_S}}} & \frac{h_{2s}(k)}{\partial b_{\rho_{N_S}}} \end{array}\right] \quad (29)$$

The appropriate partial derivatives are given in [4].

D. Initialialization

In order to perform the numerical search via ILS, an initial estimate $\hat{\boldsymbol{\theta}}^0$ is required. Assuming that the biases are null, the LOS measurements from the first and the second sensor $\alpha_1(k)$, $\alpha_2(k)$ and $\epsilon_1(k)$ can be used to solve for each initial Cartesian target position, in ECI coordinates, using (30)–(32). The two Cartesian positions formed from (30)–(32) can then be differenced to provide an approximate velocity. This procedure is analogous to two-point differencing [2] and will provide a full six-dimensional state to initialize the ILS algorithm.

$$x(k)^0 = \frac{\xi_2(k) - \xi_1(k) + \zeta_1(k)\tan\alpha_1(k) - \zeta_2(k)\tan\alpha_2(k)}{\tan\alpha_1(k) - \tan\alpha_2(k)} \quad (30)$$

$$y(k)^0 = \frac{\tan\alpha_1(k)\left(\xi_2(k) + \tan\alpha_2(k)\left(\zeta_1(k) - \zeta_2(k)\right)\right) - \xi_1(k)\tan\alpha_2(k)}{\tan\alpha_1(k) - \tan\alpha_2(k)} \quad (31)$$

$$z(k)^0 = \eta_1(k) + \tan\epsilon_1(k)\left|\frac{(\xi_1(k) - \xi_2(k))\cos\alpha_2(k) + (\zeta_2(k) - \zeta_1(k))\sin\alpha_2(k)}{\sin\left(\alpha_1(k) - \alpha_2(k)\right)}\right| \quad (32)$$

III. Simulations

In this paper we used a hypothetical scenario to test our new methodology. The missile and satellite trajectories are generated using System Tool Kit (STK). The sensor satellites are in a circular orbits of 600 km and 700 km altitude with 0°, 60° degrees inclination, respectively. The target modeled represents a long range ballistic missile with a flight time of about 20 minutes. STK provides the target and sensor positions in three dimensional Cartesian coordinates at 1 s intervals. The measurement noise standard deviation σ_s (identical across sensors for both azimuth and elevation measurements, $\sigma_s^\alpha = \sigma_s^\epsilon = \sigma_s$) was assumed to be 30 μrad. The target launch time was chosen so that the satellite sensors were able to follow the missile trajectory throughout its flight path. As shown in Figure 3, these satellite orbits enabled maximum visibility of the missile trajectory from multiple angles. The missile and satellite trajectories displayed in Figure 3 represent 5 minutes of flight time. In order to establish a baseline for evaluating the performance of our method, we also ran the simulations without biases and with biases, but without bias estimation. As discussed in the previous section, the three sensor biases were roll, pitch and yaw angle offsets. Table I summarizes the bias values (in mrad).

TABLE I
SENSOR BIASES (MRAD).

	ψ	ρ	ϕ
Sensor 1	5.7596	4.3633	-3.8397
Sensor 2	4.8869	5.4105	-5.0615

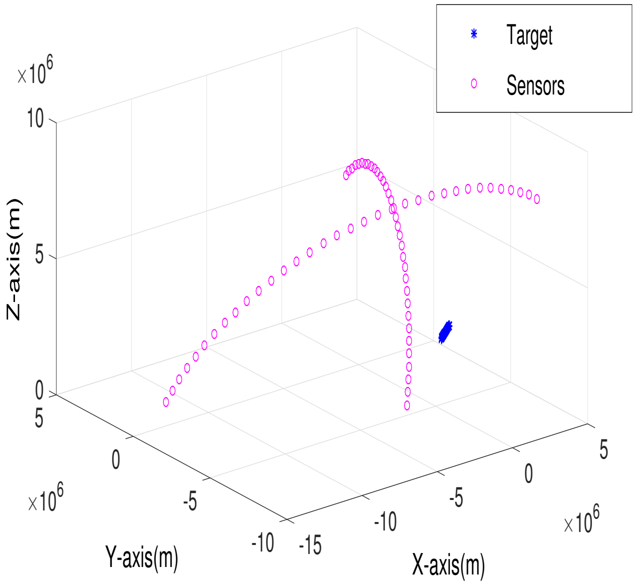

Fig. 2. Target and satellite trajectories for the two-sensor case

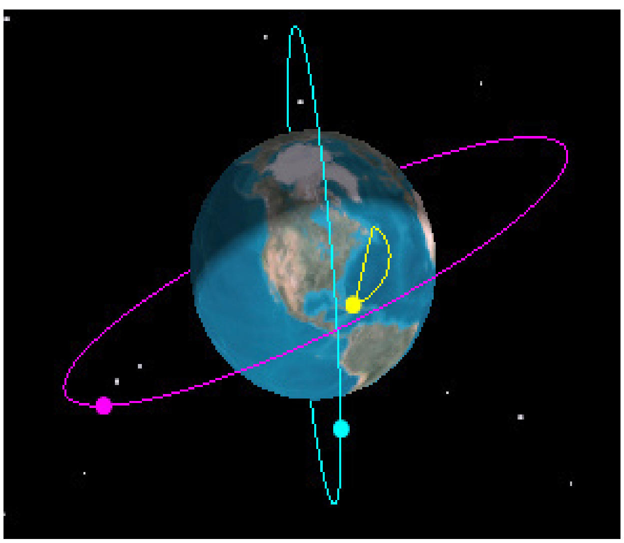

Fig. 3. Target and satellite trajectories for the two-sensor case

The RMS errors for the target position and velocity are summarized in Table II. In this table, the first estimation scheme was established as a baseline using bias-free LOS measurements to estimate the target position and velocity. For the second scheme, we used biased LOS measurements but we only estimated target position and velocity. In the last scheme, we used biased LOS measurements and we simultaneously estimated the target position, velocity, and sensor biases. Once again, bias estimation yields significantly improved target RMS position and velocity errors in the presence of biases.

TABLE II
SAMPLE AVERAGE RMSE (M) FOR THE TARGET POSITION AND VELOCITY, OVER 100 MONTE CARLO RUNS, FOR THE 3 ESTIMATION SCHEMES.

Scheme	Position RMSE	Velocity RMSE
1	107.44	5.16
2	47,161.10	25,149.32
3	494.49	19.55

IV. Conclusions and Future Work

In this paper we presented a new algorithm that uses a target of opportunity for estimation of measurement biases together with target state. The first step was formulating a general bias model for synchronized space-based optical sensors at known locations. The association of measurements is assumed to be perfect. Based on this, we used an ML approach that led to a batch nonlinear least-squares estimation problem for simultaneous estimation of the 3D Cartesian position and velocity components of the target of opportunity and the angle measurement biases of the sensors. For future work we plan to evaluate the statistical efficiency of the algorithm.

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